Dynamic Population Model for COVID-19

Prof. Alan D. Freed

J. MIKE WALKER '66 DEPARTMENT OF MECHANICAL ENGINEERING
TEXAS A&M UNIVERSITY

afreed@tamu.edu

MEEN 357, Spring 2020

2020

Overview

The COVID-19 pandemic has turned everyone's world upside down of late. Everyday we hear reports coming over our news feeds, and much of it is very scary, and rightfully so.

As this is a class in numerical methods, I thought it might be insightful for my students to have an additional viewpoint from which to interpret and filter the information that they are being bombarded with on a daily basis as we live through this tragedy.

News reports often elude to 'model predictions' upon which government officials and agencies are relying on to help them prepare for future events. In this lecture a fairly simple epidemic model is presented that may help students better understand why governments are doing what they are, and the significance of these actions.

Consider a population with N persons in it on any given day. Assign each person to one of the following groups:

- S number of *susceptible* persons to the virus.
- E number of *exposed* persons to the virus.
- I number of *infected* persons to the virus.
- R number of *recovered* persons from the virus.

We also introduce two additional groups of known dynamics:

- B number of *births* into a population.
- D number of *deaths* leaving a population due to normal causes.

Disclaimer: Your Professor is not an expert in this field.

The model presented here is the SEIR model published in:

Earn, D.J.D., Rohani, P., Bolker, B.M. and Grenfell, B.T., "A Simple Model for Complex Dynamical Transitions in Epidemics", *Science*, **287**, Jan. 2000, 667–670.

The SEIR Model

Susceptible, Exposed, Infected, Recovered

$$\frac{\mathrm{d}S}{\mathrm{d}t} = B - \left(\frac{D}{N} + \beta\frac{I(t)}{N}\right) S(t)$$
$$\frac{\mathrm{d}E}{\mathrm{d}t} = \beta\frac{I(t)}{N} S(t) - \left(\frac{D}{N} + \phi\right) E(t)$$
$$\frac{\mathrm{d}I}{\mathrm{d}t} = \phi E(t) - \left(\frac{D}{N} + \mu + \frac{1}{\tau}\right) I(t)$$
$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{1}{\tau} I(t) - \frac{D}{N} R(t)$$

Summing up the right-hand sides results in the population flux

$$\Delta N = B - D - \mu I \quad \because \quad N = S + E + I + R$$

This is a model in four variables (S, E, I, R) expressed in terms of six model parameters $(B, D, \beta, \phi, \mu, \tau)$.

Within a population of size N, these factors are considered:

- B number of births per day.
- D number of deaths per day not caused by the virus.
- β number of person-to-person contacts per person per day.
- ϕ fraction of exposed persons that become infected per day.
- μ fraction of infected persons that die per day (morbidity).
- τ average number of days needed to recover from infection.

Deaths D are distributed over the four populations: S, E, I, R.

Important factors not considered here include: the migration of persons into or out of a population, quarantining exposed and infected individuals, and the effect of vaccination, because there isn't one.

The number of exposed persons $E(t)$ increases by that fraction $I(t)/N$ of infected persons which interact with those that are susceptible $S(t)$ through parameter β, which denotes the average number of person-to-person contacts made per day. Taken away from this is that fraction of people who die from natural causes D/N plus that fraction of those exposed who contract the virus ϕ, all of which modulate the exposed $E(t)$.

The number of susceptible persons $S(t)$ increases by the births per day B, and decreases by the deaths per day D/N within population $S(t)$, less those that get moved into the exposed group via person-to-person contact, as specified by $\beta(I(t)/N)$, which also modulates the number of susceptible persons $S(t)$.

The number of infected persons $I(t)$ increases by some fraction ϕ of those who are exposed $E(t)$ per day, i.e., not all those who get exposed contract the virus. Removed from this infected population are those that die from natural causes D/N, those who succumb to the virus μ, a.k.a. the morbidity, and those who have been ill for τ days and hence recovered, all of which modulate the number of infected persons $I(t)$.

The number of recovered persons $R(t)$ increases by those who have recovered from the disease $I(t)/\tau$ and decreases by those who have died from normal causes D/N rationed over its population $R(t)$

These parameters are quite robust:

- B is 10.25 persons per day (The Eagle, for 2018).
- D is 8.17 persons per day (from life expectancy of 76.1).

These parameters are somewhat certain:

- τ is 14 days (based on reported values from news feeds).
- μ is 0.023 per day (based on reported statistics for U.S.)[1]

This characteristic is unknown to the best of my knowledge:

- ϕ guess: 0.1, i.e., 10% of all exposures result in disease.

This parameter can be affected through executive orders:

- β will be varied in our assessment that follows.

[1] On April 2, 2020: Worldwide, $\mu = 0.043$; Italy, $\mu = 0.104$.

Initial Conditions for the Model

For Brazos County, Texas, as of April 2, 2020

The initial conditions considered are:

- N_0 is 226,758 persons (Wikipedia, estimated 2018).
- S_0 is 226,210 (computed from $S_0 = N_0 - E_0 - I_0 - R_0 - D_0$).
- E_0 is 473 persons (number tested for COVID-19).
- I_0 is 68 persons (number of reported cases).
- R_0 is 3 persons have recovered from the virus.
- D_0 is 4 persons have passed from the virus.

Interesting facts:
As of April 2, 2020, 48% of all infections in Brazos County were under 40 years of age, and 69% of all infections came about from communicable (within population) transmissions.

Two-Step PECE Method for Numeric Integration

https://arxiv.org/abs/1707.02125; it is a free download

Take first step using Heun's method:

Predict: $\mathbf{y}^p_{n+1} = \mathbf{y}_n + h\,\mathbf{y}'_n + \mathcal{O}(h^2)$

Evaluate: $\mathbf{y}^{p\prime}_{n+1} = \mathbf{f}(x_{n+1}, \mathbf{y}^p_{n+1})$

Correct: $\mathbf{y}_{n+1} = \mathbf{y}_n + \frac{1}{2}h\big(\mathbf{y}'_n + \mathbf{y}^{p\prime}_{n+1}\big) + \mathcal{O}(h^3)$

Evaluate: $\mathbf{y}'_{n+1} = \mathbf{f}(x_{n+1}, \mathbf{y}_{n+1})$

Take remaining steps using your Professor's PECE method:

Predict: $\mathbf{y}^p_{n+1} = \frac{1}{3}\big(4\mathbf{y}_n - \mathbf{y}_{n-1}\big) + \frac{2}{3}h\big(2\mathbf{y}'_n - \mathbf{y}'_{n-1}\big) + \mathcal{O}(h^3)$

Evaluate: $\mathbf{y}^{p\prime}_{n+1} = \mathbf{f}(x_{n+1}, \mathbf{y}^p_{n+1})$

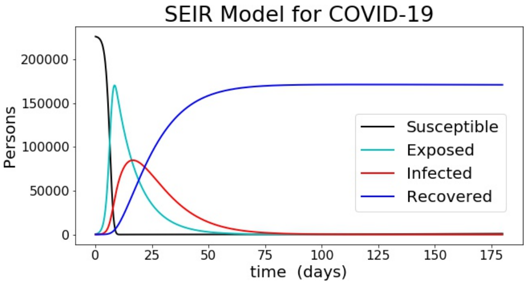
Correct: $\mathbf{y}_{n+1} = \frac{1}{3}\big(4\mathbf{y}_n - \mathbf{y}_{n-1}\big) + \frac{2}{3}h\,\mathbf{y}^{p\prime}_{n+1} + \mathcal{O}(h^3)$

Evaluate: $\mathbf{y}'_{n+1} = \mathbf{f}(x_{n+1}, \mathbf{y}_{n+1})$

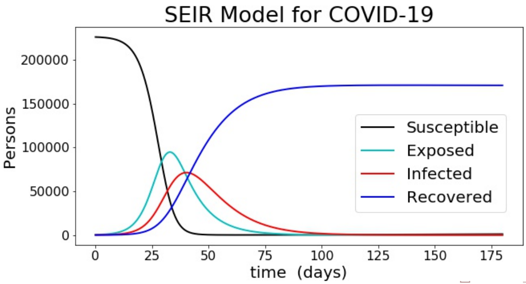
Freed, A.D., "A Technical Note: Two-Step PECE Methods for Approximating Solutions to First- and Second-Order ODEs", *arXiv:1707.02125*, July, 2017.

SEIR Predictions

Rate of transmission from exposed to infected set at $\phi = 0.1$.



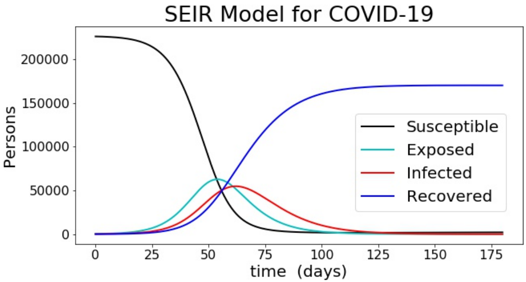
$\beta = 10$



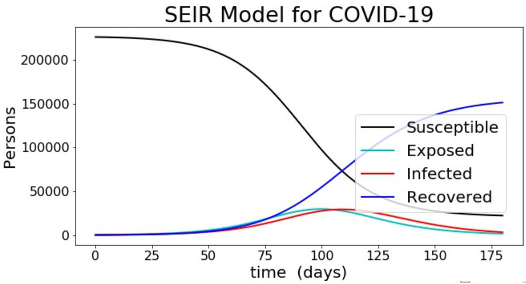
$\beta = 1$

SEIR Predictions, Continued

Rate of transmission from exposed to infected set at $\phi = 0.1$.



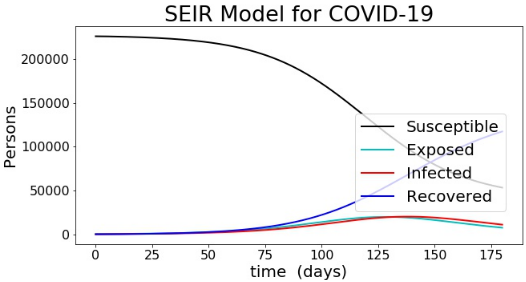
$\beta = 0.5$



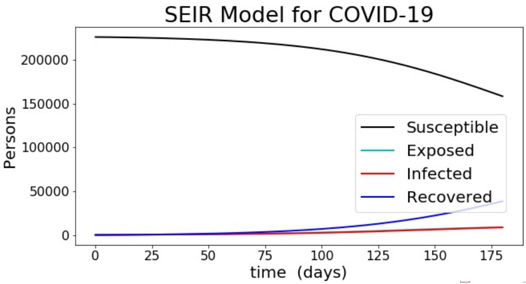
$\beta = 0.25$

SEIR Predictions, Continued

Rate of transmission from exposed to infected set at $\phi = 0.1$.



$\beta = 0.2$



$\beta = 0.15$

Spline and Integrate the Infected Response

To derive the predicted numbers for the infected and deceased, the following procedure was implemented:

1. Solve the SEIR system of equations using the PECE solver, integrating from 0 to 180 days.
2. Fit a cubic spline to the integrated infected response $I(t)$.
3. Use Romberg integration to integrate this spline from 0 to 180 days.
4. To get the number of predicted infections, take the Romberg integrated value I (number of infected days summed over the population) and divide this by τ (mean number of days needed to recover from the virus), then add the initial number of infected persons I_0 on April 2.
5. To get the number of predicted deaths, multiply the above number of predicted infections by the morbidity μ.

Six Month SEIR Predictions for the Brazos Valley

Assumes 10% transmission rate from exposed to infected.

β	infections	deaths
10	172,088	3,960
1	171,883	3,955
0.5	170,978	3,934
0.25	151,791	3,493
0.2	117,703	2,709
0.15	38,701	892
0.1	4,334	102

Parameter β specifies the average number of person-to-person contacts made per day per person within a population. This parameter has a HUGE impact on predicted deaths, which is why our government officials at the federal, state and local levels have implemented policies of social distancing, shelter in place and quarantining.

Please abide by these policies. Lives depend upon it!

Observations

What your Professor has learned from this exercise.

1. If left on its own, outcomes from this pandemic will be the same, it may take longer (i.e., ϕ is smaller and/or β is smaller) or it may happen faster (viz., ϕ is larger and/or β is larger), but the final outcomes are about the same.
2. Each person in their community must do whatever they can to minimize/remove 'risky' person-to-person interactions with other members of their community outside of their immediate family. Our federal, state and local governments are providing guidelines for us to follow.
3. The further we can push the response curves out in time (to the right) the more time we provide to our medical researchers to secure a vaccine, and the longer it will take before our healthcare providers become overwhelmed.

Disclaimer: Again, your Professor is not an expert in this field.

Thank you! Questions?

My thoughts take me back to carefree days. May yours, too.